CHAPTER-4

DOUBLY REINFORCED RECTANGULAR SECTION

4.1

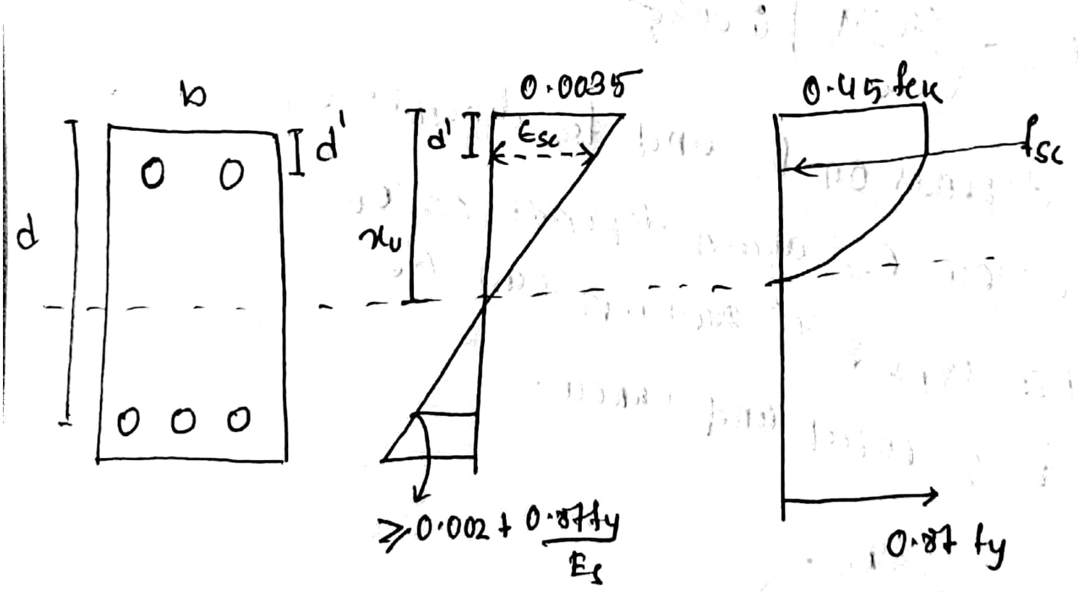
When longitudinal R/F is provided in tension as well as compression zone of section then such section is termed as doubly R/F. i

it is provided when

→ Section size is Restricted (b and D both).

→ $BM_u > M_{ulim}$ of section.

4.2 (Analysis of section)



for position of NA

$$C = T$$

$$\Rightarrow \quad C_c + C_s = T$$

$$\Rightarrow \quad 0.36 f_{ck} x_u b$$

$$\left(0.36 f_{ck} x_u b - A_{sc} \times 0.45 f_{ck}\right) + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 f_{ck} x_u b + \left(f_{sc} - 0.45 f_{ck}\right) A_{sc} = 0.87 f_y A_{st}$$

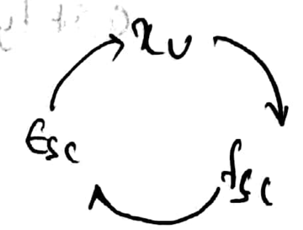
f_{sc} is required to calculate x_u, since f_{sc} depends on ϵ_{sc} so ϵ_{sc} is being calculated as follow.

for strain diagram.

$$\frac{\epsilon_{sc}}{x_u - d'} = \frac{0.0035}{x_u}$$

$$\Rightarrow \epsilon_{sc} = \left(\frac{x_u - d'}{x_u}\right) 0.0035$$

→ x_u depends on f_{sc} and f_{sc} depends on ϵ_{sc}. Now ϵ_{sc} again depends on x_u so this types of problem can be solved by trial and error.



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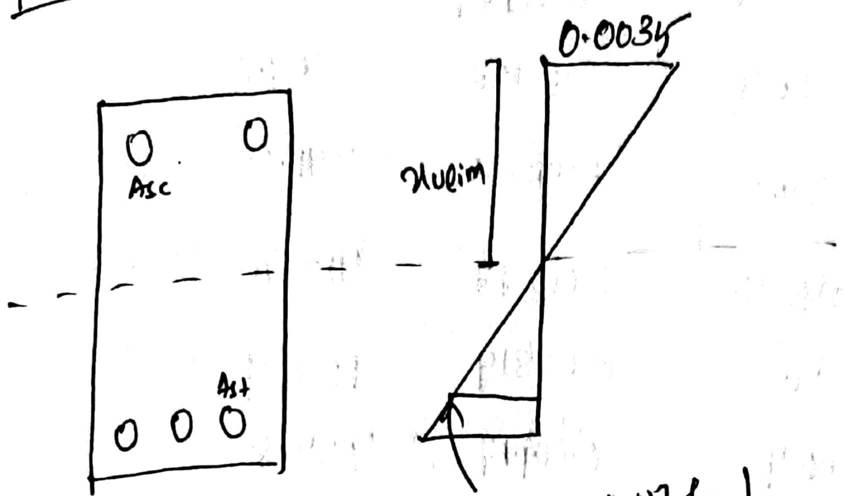
4.3 (POSITION OF NEUTRAL AXIS)

4.3.1 (balanced section)



$= \left(0.002 + \frac{0.87 f_y}{E_s}\right)$

$x_{ulim} = 0.53d$ (Fe 250)

$= 0.48d$ (Fe 415)

$= 0.46d$ (Fe 500).

4.3.2 (UNDER R/F SECTION)

Step-1

calculate x_{ulim} and assume $x_{u1} = x_{ulim}$

Step-2

calculate ϵ_{sc}

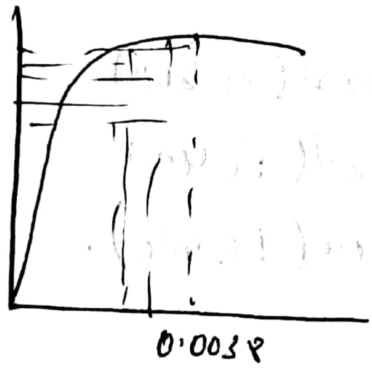
$$\epsilon_{sc} = \left(\frac{x_{u1} - d'}{x_{u1}}\right) 0.0035$$

Step-3

calculate f_{sc} from stress-strain curve of steel corresponding to ϵ_{sc}.

$f_{yd} = 0.87 f_y$

	Fe415 Strain (ϵ_{sc})	Fe415 Stress (f_{sc})	Fe500 Strain	Fe500 Stress
$0.8 f_{yd}$	0.00144	288.7	0.00174	347.8
$0.85 f_{yd}$	0.00163	306.7	0.00195	369.6
$0.90 f_{yd}$	0.00192	324.8	0.00226	391.3
$0.95 f_{yd}$	0.00241	342.8	0.00277	413.0
$0.975 f_{yd}$	0.00276	351.8	0.00319	423.9
$1.0 f_{yd}$	0.00380	360.9	0.00417	434.8



Note

→ Design yield stress $= f_{yd} = 0.87 f_y$

→ If ϵ_{sc} lies between values of above table then f_{sc} is calculated by linear interpolation.

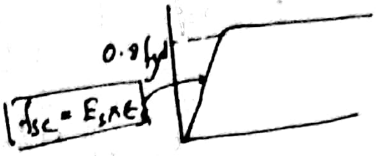
$x_1 \rightarrow y_1$

$\epsilon_{sc} \rightarrow f_{sc}$

$x_2 \rightarrow y_2$

$$f_{sc} = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)} (\epsilon_{sc} - x_1)$$

→ for ϵ_{sc} less than 0.00144 and 0.00174 of Fe415 and Fe500 respectively, f_{sc} is calculated by multiplying E_s to ϵ_s. [as ~~below~~ up to $0.8 f_{yd}$ stress-strain curve is straight]



Step-IV

calculate ~~the~~ position of NA

$$C = T$$

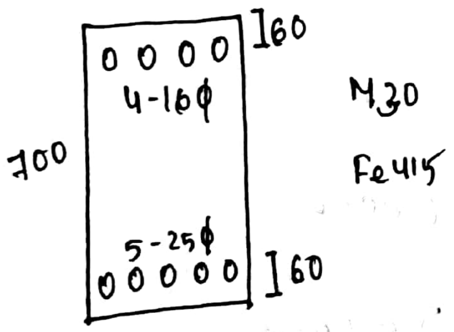
$$\Rightarrow 0.36 f_{ck} x_u b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = ??$$

Step-V

if x_u of step-4 $\neq$ x_u of step-1, then repeat step-1 to step-4 with $x_{u2} = x_u$ of step 4

Q calculate the position of NA



First trail

① $x_{u1} = x_{ulim} = 0.48d = 0.48 \times 640 = 307.2 mm$

② $\epsilon_{sc} = \left(\frac{x_{u1} - d'}{x_u}\right) 0.0035$

$= \left(\frac{307.2 - 60}{307.2}\right) 0.0035$

$= 0.00281$

3) $f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1)$

$= 351.8 + \left(\frac{360.9 - 351.8}{0.0038 - 0.00276}\right)(0.00281 - 0.00276)$

$= 352.23\ N/mm^2$

(4) $C = T$

$0.36 f_{ck} x_u b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$

$\Rightarrow 0.36 \times 30 \times x_u \times 360 + (352.33 - 0.45 \times 30) \times \frac{\pi}{4} \times 16^2 \times 4$

$= 0.87 f_y \times 5 \times \frac{\pi}{4} \times 25^2$

$x_u = 157.85 mm \neq x_{u1}$

so <u>2nd trail</u>

(1) $x_{u2} = 157.85\ mm$

(2) $\epsilon_{sc} = \left(\frac{x_{u2} - d'}{x_{u2}}\right) 0.0035$

$= \left(\frac{157.85 - 60}{157.85}\right) 0.0035$

$= 0.00217$

(3) $f_{sc} = 324.8 + \left(\frac{342.8 - 324.8}{0.00241 - 0.00192}\right)$

$(0.00217 - 0.00192)$

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= 333.98 N/mm²

(4) c = 1

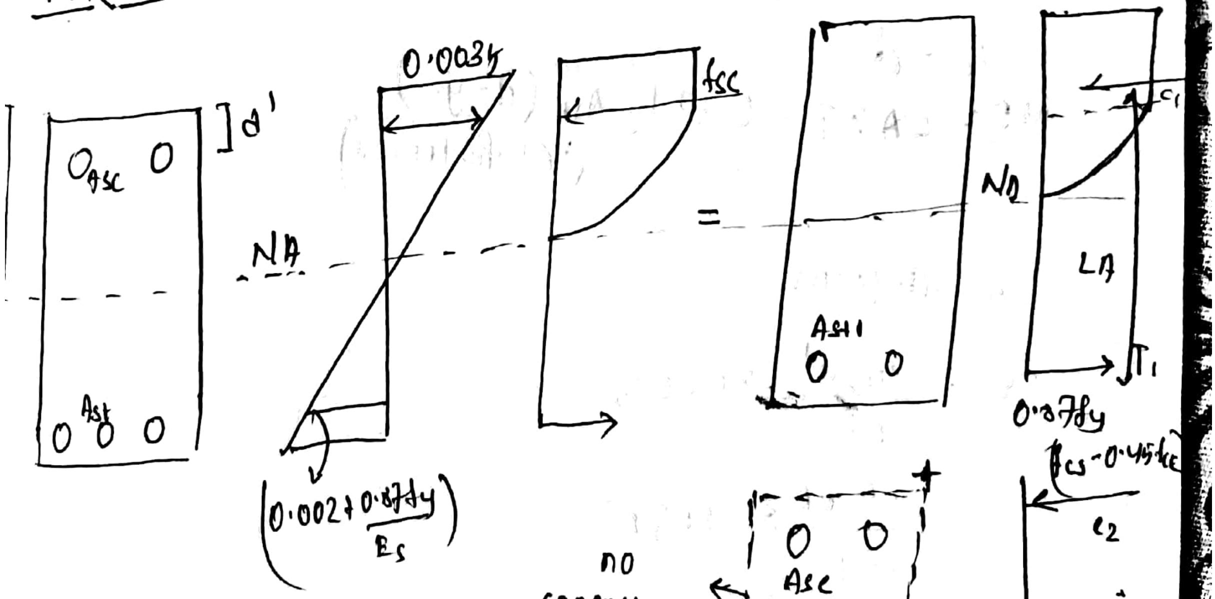
$0.36 f_{ck} x_u b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$

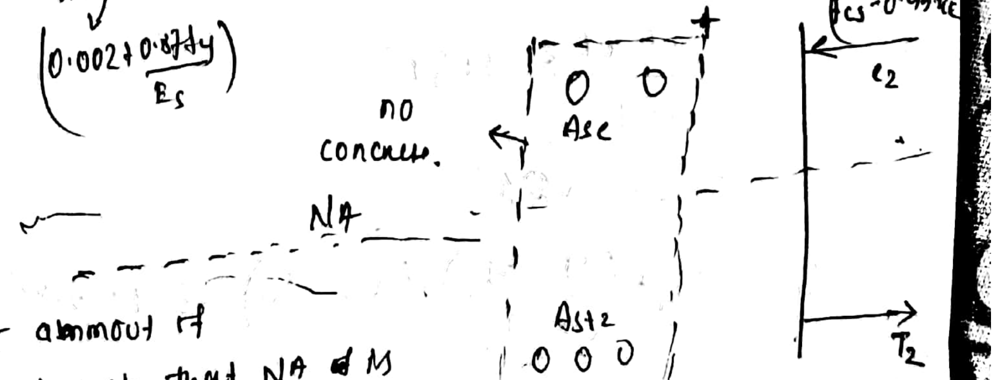
$\Rightarrow 0.36 \times 30 \times x_u \times 360 + (333.98 - 0.45 \times 30)$

$\Rightarrow x_u = 161.63 \text{ mm} \neq x_{u2}$

few more trials are required for relatively closer to exact value of x_u. in this case considering $x_u = 159$ mm as an approximate value.

MR OF SECTION





Ast1 is that amount of steel is provided such that NA & M same as original. (Balanced Section)

$MR = MR_1 + MR_2$

$= C_1 \times LA_1 + C_2 \times LA_2$

$= 0.36 f_{ck} x_u b (d - 0.42 x_u)$

$+ (f_{sc} - 0.45 f_{ck}) A_s (d - d')$

MR from tension side.

position of net comp force

from extreme comp fiber (from Top).

$$\bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$$

$y_1 = 0.42 x_u$

$y_2 = d$

$MR = LA \times T = 0.87 f_y A_{st} (d - \bar{y})$ (not preferred)

from previous question.

$x_u = 159mm$

$$\epsilon_{sc} = \frac{x_u - d'}{x_u} \times 0.0035$$

$$= \frac{159 - 60}{159} \times 0.0035$$

$= 0.00218$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\epsilon_{sc} - x_1) = 334.35 N/mm$$

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$MR = MR_1 + MR_2$

$= C_1 \times LA + C_2 \times LA$

$= 0.36 f_{ck} x_u b (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_s (d - d')$

$= 0.36 \times 30 \times 159 \times 360 (640 - 0.42 \times 159) + (334.35 - 0.45 \times 30) 4 \times \frac{\pi}{4} \times 16^2 (640 - 60)$

$= 504.02$ kNm.

4.5 (DESIGN OF DOUBLY REINFORCED RECTANGULAR SECTION)

Doubly reinforced sections are provided when section size is restricted, so size of section is known. Now Asc and Ast are to be calculated.

Step-1

Calculate Factored / ultimate BM.

Step-2

Calculate Mulim of given section.

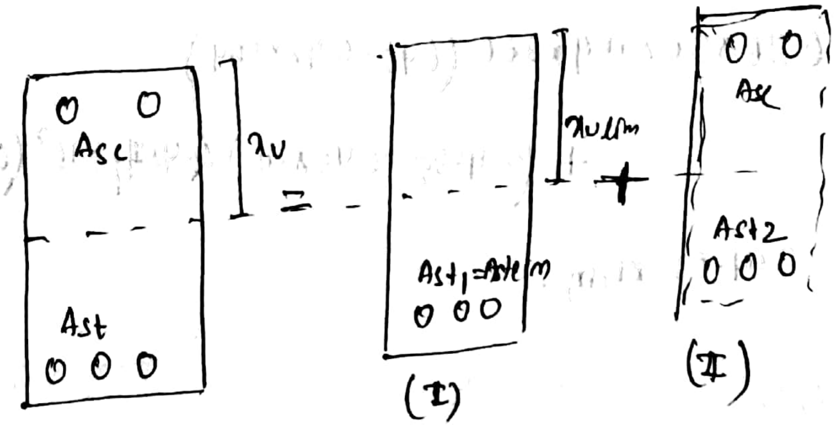
$M_{ulim} = Q bd^2$

Step-3

if $BM_u \leq M_{ulim}$ → Then section is designed as singly Reinforced.

So $A_{st} = \frac{0.5 f_{ck} bd}{f_y}\left(1-\sqrt{1-\frac{4.6 BM_u}{f_{ck} bd^2}}\right)$

if $BM_u > M_{ulim}$ → Then section is designed as doubly reinforced balanced section.



Step-4

Calculate Ast1

$C_1 = T_1$

$0.36 f_{ck} x_{ulim} b = 0.87 f_y A_{st1}$

$\Rightarrow A_{st1} = ??$

Step-5

Calculate BM taken by (II)

$BM_u = MR$

$\Rightarrow BM_u = MR_1 + MR_2$

$\Rightarrow BM_u = MR_{lim} + MR_2$

$\Rightarrow BM_u - MR_{lim} = MR_2$

$\Rightarrow MR_2 = BM_u - MR_{lim}.$

Step-6

Calculate A_{st2}

$$MR_2 = T_2 \times LA_2$$

$$(BM_u - M_{ulim}) = 0.87 f_y A_{st2} (d - d')$$

$\Rightarrow A_{st2} = ??$

Step-7

Calculate A_{sc}

$$C_2 = T_2$$

$$(f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st2}$$

$\Rightarrow A_{sc} = ??$

Step-8

A_{st} and A_{sc} calculated above should be within permissible limit.

→ $\frac{A_{st\,min}}{bd} > \frac{0.85}{f_y}$

→ $A_{st\,max} = 4\%$ of gross Area.
$= 0.04 \times (b \times D)$

Note
we should not increase more than 4% because difficult in compaction of concrete.

→ $A_{sc\,min}$ = calculated A_{sc}

→ $A_{sc,max}$ = 4% of gross Area = $0.04\,bD$

Q Design a section for factored BM 110 KNm it section size is restricted to (250×380) mm effective cover = 50 mm, M20, Fe415.

① BM_u = 110 KNm

② $M_{ulim} = 0.138 f_{ck} bd^2$

$= 0.138 \times 20 \times 250 \times 330^2$

$= 75.14$ KNm

③ Since $BM_u > M_{ulim}$ so section is designed as doubly R/F balanced section

③ $x_u = x_{ulim} = 0.48\, d = 0.48 \times 330$

$= 158.4$ mm

④ For A_{st1}

$C_1 = T_1$

$\Rightarrow 0.36 f_{ck} x_{ulim} b = 0.87 f_y A_{st_1}$

$\Rightarrow A_{st1} = 789.69\ mm^2$

⑤ $MR_2 = BM_u - M_{ulim}$

$= 110 - 75.14$

$= 34.86$ KNm.

⑥ A_{st2}

$MR_2 = T_2 \times LA_2$

$MR_2 \Rightarrow 0.87\, f_y\, A_{st2}\, (d-d')$

$\Rightarrow 34.86 \times 10^6 = 0.87 \times 415 \times A_{st2} (330-50)$

$A_{st2} = 344.82\ mm^2$

7) For Asc

$$\varepsilon_{sc} = \left(\frac{x_{ulim} - d'}{x_{ulim}}\right) 0.0035$$

$$= \left(\frac{158.4 - 50}{158.4}\right) 0.0035$$

$$= 0.00239.$$

$$f_{sc} = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\varepsilon_{sc} - x_1)$$

$$= 342.06\ N/mm^2$$

$C_2 = T_2$

$\Rightarrow (f_{sc} - 0.45 f_{ck}) A_s = 0.87\, f_y\, A_{st2}$

$(342.06 - 0.45 \times 20) A_{sc} = 0.87 \times 415 \times 344.82$

$A_{sc} = 373.79\ mm^2$

(7) $A_{st} = A_{st1} + A_{st2} = 1134 \cdot 72 \ mm^2$

$A_{sc} = 373 \cdot 79 \ mm^2$

providing

$A_{st} = 3-22\phi$

$A_{sc} = 2-16\phi$ ✗

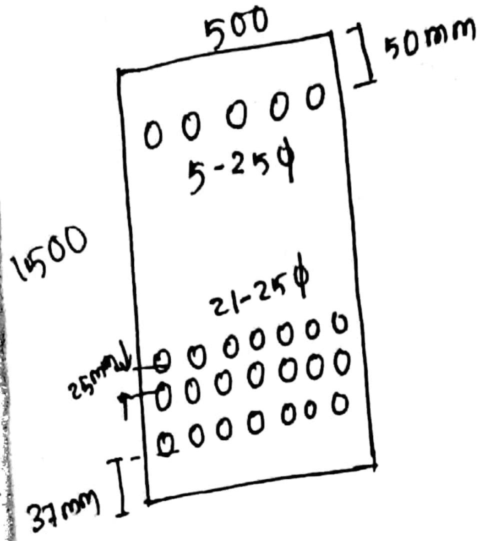
$2-20\phi$ ✓

compression steel is increased by higher ammount than increase in tension steel to keep section under R/F.

Q A SS beam of effective span is 18m is reinforced as given below. (M25 and Fe415)

(1) Find max^m BM at limit state of collapse if stress in compression reinforcement is $0.9566 f_{yd}$

(2) What shall be max^m super imposed working live load can be applied on the beam.



① FOR portion of NA C=T

$C_c + C_s = T$

$\Rightarrow 0.36 f_{ck} x_u b + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$

$\Rightarrow 0.36 \times 25 \times x_u \times 500 + (0.95667 \times 0.87 \times 415 - 0.45 \times 25)$

$= 0.87 f_y$

$\Rightarrow x_u = 644.83$ mm

d = distance b/w extreme comp fiber to cg of tension R/F

$= D - \left(\text{clear cover} + \text{dia of bar} + \text{gap} - \frac{\text{dia of bar}}{2}\right)$

$= 1500 - \left(37.5 + 25 + 25 + \frac{25}{2}\right)$

$= 1400$ mm.

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$x_{ulim} = 0.48d$

$= 0.48 \times 1400$

$= 672 \text{ mm}$

Since $x_u < x_{ulim}$ so section is under R/F.

$MR = MR_1 + MR_2$

$= C_1 LA_1 + C_2 LA_2$

$MR = 0.36 f_{ck} x_u b (d - 0.42 x_u)$

$+ (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d')$

$= 0.36 \times 25 \times 644.8 \times 500 (1400 - 0.42 \times 644.8)$

$+ (0.9566 \times 0.87 \times 415 - 0.45 \times 25)$

$5 \frac{\pi}{4} (25)^2 (1400 - 50)$

$= 4383.7 \text{ kNm}$.

(2) $DL = 0.5 \times 1.5 \times 1 \times 25 = 18.75 \text{ kN/m}$

$LL = W$

factored total load $= 1.5 (W + 18.75)$

$BM_{max} = \frac{WL^2}{8}$

$= \frac{1.5 (W + 18.75) \times 18^2}{8}$

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for max^m superimposed LL.

$BM_{max} = MR$

$\Rightarrow \frac{15(w+18.75) \times 18^2}{8} = 4383.7$

$\Rightarrow w = 53.47 kN/m$

Note

→ if reduction in concrete are due to presense of compression in steel is not considered then term $(0.45 f_{ck} A_{sc})$ is removed from all expression.